A Little Higgs model of neutrino masses

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Abstract

Little Higgs models are formulated as effective theories with a cut-off of up to 100 times the electroweak scale. Neutrino masses are then a *puzzle*, since the usual *see-saw* mechanism involves a much higher scale that would introduce quadratic corrections to the Higgs mass parameter. We propose a model that can *naturally* accommodate the observed neutrino masses and mixings in Little Higgs scenarios. Our framework does not involve any large scale or suppressed Yukawa couplings, and it implies the presence of three extra (Dirac) neutrinos at the TeV scale. The masses of the light neutrinos are induced radiatively, they are proportional to small ($\approx$ keV) mass parameters that break lepton number and are suppressed by the Little Higgs cut-off.

Introduction. The stability of the electroweak (EW) scale at the loop level has been the main motivation to search for physics beyond the standard model (SM) during the past 30 years. Namely, SM loops introduce quadratic corrections to the EW scale. If this scale is *natural*, consistent with the dynamics and not the result of an accidental cancellation between higher scales, then new physics must compensate the SM quadratic contributions. In particular, top quark corrections to the Higgs mass squared become of order $(500 \text{ GeV})^2$ for a cutoff ≈ 2 TeV, what would suggest new contributions (from supersymmetry, extra dimensions,...) of the same order at the reach of the LHC.

Little Higgs (LH) ideas [1, 2, 3] provide another very interesting framework with a scalar sector free of unsuppressed loop corrections. New symmetries protect the EW scale and define consistent models with a cutoff as high as $\Lambda \approx 10$ TeV. Therefore, these models could describe all the physics to be explored in the next generation of accelerators. More precisely, in LH models the scalar sector has a (tree-level) global symmetry G that is broken spontaneously at a scale $f \approx 1$ TeV. The SM Higgses are then Goldstone bosons (GBs) of the broken symmetry, and remain massless and with a flat potential at that scale. Yukawa and gauge interactions break explicitly the global symmetry. However, the models are built in such a way that the loop diagrams giving non-symmetric contributions must contain at least two different couplings. This *collective* breaking keeps the Higgs sector free of quadratic top-quark and gauge contributions. At the same time, loops (and/or explicit non-symmetric terms in the scalar potential) give mass and quartic couplings to the Higgs, both necessary to break the SM symmetry (see [4] for a recent review).

The inclusion of neutrino masses in this framework looks problematic. In the SM these masses require a new scale much larger than the EW one,

$$\mathcal{L}_{eff} = \frac{1}{2\Lambda_\nu} h^\dagger h^\dagger LL + \text{h.c.} , \tag{1}$$

where $h = (h^0 \ h^-)$ and $L = (\nu \ e)$ are, respectively, the SM Higgs and lepton doublets. At the EW phase transition $\langle h^0 \rangle = v$ and the neutrinos get their mass $m_\nu = v^2/\Lambda_\nu \approx 0.1$ eV, what implies $\Lambda_\nu \approx 10^{14}$ GeV. This effective (low-energy) scenario is simply realised using the *see-saw* mechanism [5]. A SM singlet per family, n^c, is introduced with a large Majorana mass M and sizeable Yukawa couplings λ_ν with the lepton doublets:

$$\mathcal{L}_\nu = \lambda_\nu h^\dagger L n^c + \frac{1}{2} M n^c n^c + \text{h.c.} \tag{2}$$

(the fields denote two-component left-handed spinors and family indices are omitted). The spectrum is then three (Majorana) fields of mass $\approx M$ plus the observed low-energy neutrinos with mass $m_\nu \approx \lambda_\nu^2 v^2/M$.

This scenario looks inconsistent in LH models, since the EW scale is not stable at the quantum level. In particular, the diagram in Fig. 1 gives a large contribution to the Higgs mass proportional to M^2:

$$\Delta m_h^2 \approx -\frac{\lambda_\nu^2}{8\pi^2}\left(C_{UV} + \frac{1}{2}M^2\left(1 - 2\log M^2\right)\right) , \tag{3}$$

where we have used dimensional regularization and C_{UV} contains the ultraviolet divergence and renormalization-scale dependence. In principle LH models would avoid this type of corrections from heavy fields just because they are supposed to be effective theories valid only below a cutoff $\Lambda \approx (4\pi)^2 v$. However, if the *see-saw* scale M is at (or below) the cut-off Λ, neutrino masses would be unacceptably large. Therefore, the problem would be how to generate the operator in Eq. (1) without introducing also a term $\Delta m_h^2 h^\dagger h$ of order M^2 in the low-energy Lagrangian. The option of not using the *see-saw* mechanism and assume that neutrino masses are not different in origin from the masses of quarks and leptons requires $M \approx 0$ and $\lambda_\nu \approx 10^{-12}$, a number that would demand an explanation (see below). Other scenarios for neutrino masses [6, 7] which do not involve a mass $M \gg 1$ TeV could be consistent with LH ideas. For example, the model in [8] has $M \approx 1$ TeV and extra scalars that give masses to neutrinos but not to quarks and charged leptons. The symmetries and structure of this model (the scalars get VEVs of order MeV), however, seem difficult to accommodate in a simple LH model. Here we propose a LH framework for neutrino masses that does not require large masses nor suppressed Yukawa couplings. It involves a quasi-Dirac field per family at the TeV scale and small ($\approx$ keV) Majorana mass terms breaking lepton number. The SM neutrinos are then Majorana fields that get their masses and mixings at the loop level.

Cancellations in Little Higgs models. LH models are able to explain why the top quark does not introduce one-loop quadratic corrections to m_h^2. In all the cases this is achieved extending the global symmetries to the quark sector, so that the third generation
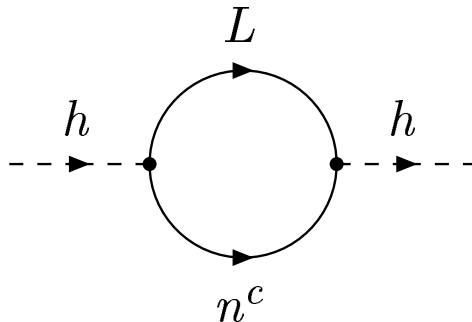

Figure 1: Diagram introducing corrections of order M^2 to m_h^2.

appears in multiplets of $SU(3)$. In particular, the doublet $Q = (t \, b)$ becomes a triplet Ψ_Q. In the same way, in order to build a consistent model of neutrino masses the global symmetry will be extended and the lepton doublets will become triplets.

Let us focus on the *simplest* LH model [9, 10], although our arguments would be analogous in the original *littlest* Higgs model [3] or other more complicated scenarios [11, 12]. Here the scalar sector contains two triplets, ϕ_1 and ϕ_2, of a global $SU(3)_1 \times SU(3)_2$ symmetry:

$$
\begin{aligned}
\phi_1 &\rightarrow e^{i\theta_1^a T^a} \phi_1 \, , \\
\phi_2 &\rightarrow e^{i\theta_2^a T^a} \phi_2 \, ,
\end{aligned}
\tag{4}
$$

where T^a are the generators of $SU(3)$. To get gauge interactions, the diagonal combination of the two $SU(3)$ is made local:

$$
\phi_{1(2)} \rightarrow e^{i\theta^a(x) T^a} \phi_{1(2)} \, .
\tag{5}
$$

At the scale f the scalar triplets get vacuum expectation values (VEVs) and break the global symmetry to $SU(2)_1 \times SU(2)_2$. For simplicity, it is usually assumed identical VEVs for both triplets

$$
\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \, , \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \, .
\tag{6}
$$

The initial 12 scalar degrees of freedom in the two triplets contain 10 GBs plus two massive fields. However, these VEVs also break the gauge symmetry $SU(3) \times U(1)_\chi$ to $SU(2)_L \times U(1)_Y$, a process that will absorb 5 of the GBs. All this becomes apparent if the two triplets are parameterized

$$
\begin{aligned}
\phi_{1(2)} = \quad &\exp\left\{ \frac{i}{f} \begin{pmatrix} & h' \\ h'^\dagger & \eta' \end{pmatrix} \right\} \times \\
&\exp\left\{ +(-)\frac{i}{f} \begin{pmatrix} & h \\ h^\dagger & \eta \end{pmatrix} \right\} \begin{pmatrix} 0 \\ f + \frac{r_{1(2)}}{\sqrt{2}} \end{pmatrix} \, ,
\end{aligned}
\tag{7}
$$

where h' and h are (complex) $SU(2)$ doublets and η', η, r_1 and r_2 are (real) $SU(2)$ singlets. At the scale f h' and η' are *eaten* by the massive vector bosons of $SU(3)$, $r_{1,2}$ get massive, and h (and possibly η) are the SM Higgses. Therefore

$$
\begin{aligned}
\phi_{1(2)} \approx \quad &\begin{pmatrix} 0 \\ f + \frac{r_{1(2)}}{\sqrt{2}} \end{pmatrix} +(-) \, i(1 + \frac{r_{1(2)}}{f\sqrt{2}}) \begin{pmatrix} h \\ \eta \end{pmatrix} \\
&- \frac{1}{2}(1 + \frac{r_{1(2)}}{f\sqrt{2}}) \begin{pmatrix} \eta h \\ h^\dagger h + \eta^2 \end{pmatrix} \, .
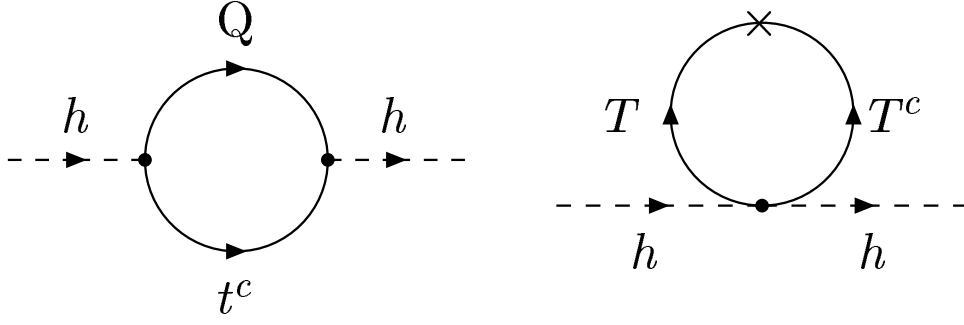\end{aligned}
\tag{8}
$$

4

Figure 2: One-loop corrections to m_h^2 in LH models.

In this model the top-quark Yukawa sector includes a triplet $\Psi_Q = (Q\ T)$ and two singlets (t_1^c, t_2^c), and it is described by the Lagrangian

$$
\begin{aligned}
\mathcal{L}_t &= \lambda_1 \phi_1^\dagger \Psi_Q t_1^c + \lambda_2 \phi_2^\dagger \Psi_Q t_2^c + \text{h.c.} \\
&\supset \lambda_t (h^\dagger Q t^c + f T T^c - \frac{1}{2f} h^\dagger h T T^c) + \text{h.c.} ,
\end{aligned}
\tag{9}
$$

where $t^c = (i/\sqrt{2})(t_2^c - t_1^c)$, $T^c = (1/\sqrt{2})(t_2^c + t_1^c)$, and we have taken $\lambda_1 = \lambda_2 = \lambda_t/\sqrt{2}$. The one-loop quadratic corrections in Fig. 2 cancel, which reflects that if one of the $\lambda_{1,2}$ couplings is zero the global $SU(3)_1 \times SU(3)_2$ symmetry would be exact in this sector.

Analogously, the model includes one lepton triplet $\Psi_L = (-iL\ N)$ (we use the $-i$ phase to simplify the couplings) and one singlet n^c per generation [13]. The Lagrangian

$$
\begin{aligned}
\mathcal{L}_\nu &= \lambda_\nu \phi_1^\dagger \Psi_L n^c + \text{h.c.} \\
&\supset \lambda_\nu (-h^\dagger L n^c + f N n^c - \frac{1}{2f} h^\dagger h N n^c) + \text{h.c.}
\end{aligned}
\tag{10}
$$

respects the global symmetries and does not generate one-loop quadratic corrections to m_h^2. When the Higgs h gets a VEV $v = 174$ GeV and breaks the EW symmetry, the Yukawa couplings induce mass terms and define the matrix

$$
\begin{array}{ccc}
\nu & N & n^c \\
\begin{pmatrix}
0 & 0 & -\lambda_\nu v \\
0 & 0 & \lambda_\nu f \\
-\lambda_\nu v & \lambda_\nu f & 0
\end{pmatrix}
&
\begin{array}{c}
\nu \\
N \\
n^c
\end{array}
\end{array}
\tag{11}
$$

The neutrino sector will then contain three Dirac fields of mass $\approx \lambda_\nu f$ plus three massless neutrinos $\nu' \approx \nu + v/f\, N$. Several observations are here in order.

(i) The SM neutrinos are exactly massless at this stage. To give them masses one would need extra singlets (which could combine with them to define Dirac neutrinos) and/or

5

new terms that break lepton number (which could introduce Majorana masses for the SM neutrinos).

(ii) The neutrino sector in Eq. (10) does not break the $SU(3)_1 \times SU(3)_2$ symmetry: both ϕ_1 and Ψ_L are triplets of $SU(3)_1$ and singlets of $SU(3)_2$. At the loop level λ_ν will contribute to the (invariant) operator $\phi_1^\dagger \phi_1$ in the scalar potential, but it will not introduce quadratic corrections to m_h^2 (which would require a symmetry-breaking operator $\phi_1^\dagger \phi_2$). Quadratic corrections to m_h^2 would appear at one loop if we add to the Lagrangian a term $\lambda_\nu' \phi_2^\dagger \Psi_L n^c$, and at higher order if we combine λ_ν with other Yukawa and gauge couplings.

(iii) Any realistic LH model needs a mass term $-m_h^2 \phi_1^\dagger \phi_2$ and a quartic coupling $\lambda(\phi_1^\dagger \phi_2)^2$ to trigger EW symmetry breaking. These terms may appear at the loop level, from scalar couplings with the global symmetry-breaking sectors of the model [12]. Once these terms are included, higher order corrections will induce the terms $\lambda_\nu' \phi_2^\dagger \Psi_L n^c$ in the Lagrangian.

A mechanism for neutrino masses in LH models. To obtain massive SM neutrinos we must then introduce additional singlets or break lepton number. It is easy to see that the first possibility requires very suppressed Yukawa couplings. An extra singlet (per generation) would make this sector identical to the top-quark sector in Eq. (9), with two Dirac fields of masses proportional to f and v. The SM neutrinos would then be too heavy unless the corresponding Yukawa couplings are very suppressed. This escenario could be naturally realized in models with extra dimensions [14, 15] or in holographic models [10], where the Higgs appears as a composite particle of some strongly coupled dynamics.

Therefore, the scenario that we propose requires just one singlet n^c per family and the breaking of lepton number. The simplest way to parameterize this breaking is through a Majorana mass for n^c, so we add the term $\frac{1}{2} M n^c n^c + \text{h.c.}$ in the Lagrangian in Eq. (10). At lowest order the neutrino mass matrix would read

$$
\begin{array}{ccc}
\nu & N & n^c
\end{array}
$$
$$
\begin{pmatrix}
0 & 0 & -\lambda_\nu v \\
0 & 0 & \lambda_\nu f \\
-\lambda_\nu v & \lambda_\nu f & M
\end{pmatrix}
\begin{array}{c}
\nu \\
N \\
n^c
\end{array}
. \tag{12}
$$

This matrix implies two massive states and one massless neutrino per generation. The massless field, however, will get a mass at the loop level. The diagram if Fig. 3 generates terms like

$$
\begin{aligned}
\mathcal{L}_1 &= \frac{1}{2\Lambda_\nu} (\phi_2^\dagger \Psi_L)(\phi_2^\dagger \Psi_L) + \text{h.c.} \\
&\supset \frac{1}{2\Lambda_\nu} (h^{0\dagger}\nu + fN)(h^{0\dagger}\nu + fN) + \text{h.c.}
\end{aligned} \tag{13}
$$

that will induce masses for the SM neutrinos. If $M < f$ and $m_h^2 < (\lambda_\nu f)^2$ and assuming diagonal couplings we obtain

$$\frac{1}{\Lambda_\nu} \approx \frac{\lambda M}{16\pi^2 f^2} \frac{x - 1 - \log x}{(x-1)^2} \ , \tag{14}$$

where $x \equiv m_h^2/(\lambda_\nu f)^2$ and λ is the Higgs quartic coupling. Notice that $1/\Lambda_\nu$ vanishes if $M = 0$, since the term in Eq. (13) breaks lepton number. Combined with the terms in Eq. (10) it also breaks the global symmetries (it is proportional to the Higgs quartic coupling λ). Although the mechanism to generate λ may be different in each LH model, λ must be $\approx 2m_h^2/v^2$.

The complete neutrino mass matrix is then

$$\begin{array}{ccc} \nu & N & n^c \end{array}$$
$$\begin{pmatrix} v^2/\Lambda_\nu & vf/\Lambda_\nu & -\lambda_\nu v \\ vf/\Lambda_\nu & f^2/\Lambda_\nu & \lambda_\nu f \\ -\lambda_\nu v & \lambda_\nu f & M \end{pmatrix} \begin{array}{c} \nu \\ N \\ n^c \end{array} \ . \tag{15}$$

Its diagonalization gives two heavy neutrinos, n^c and $N' \approx N - v/f \ \nu$ (they define a quasi Dirac field), of mass $\lambda_\nu f$ plus a SM neutrino $\nu' \approx \nu + v/f \ N$ of mass (in the limit $m_h^2 \ll (\lambda_\nu f)^2$)

$$m_\nu \approx 4v^2/\Lambda_\nu \approx v^2 \frac{\lambda M}{4\pi^2 f^2} \log \frac{(\lambda_\nu f)^2}{m_h^2} \tag{16}$$

per family. To obtain $m_\nu \approx 0.1$ eV the Majorana mass M must be ≈ 0.1 keV.

A first interesting consequence of this result (that distinguishes our framework from other scenarios for neutrino masses) is that the light masses tend to depend only logarithmically on the Yukawa couplings. Any difference or hierarchy in the Yukawa sector will appear *softened* by the logarithm in the neutrino masses.
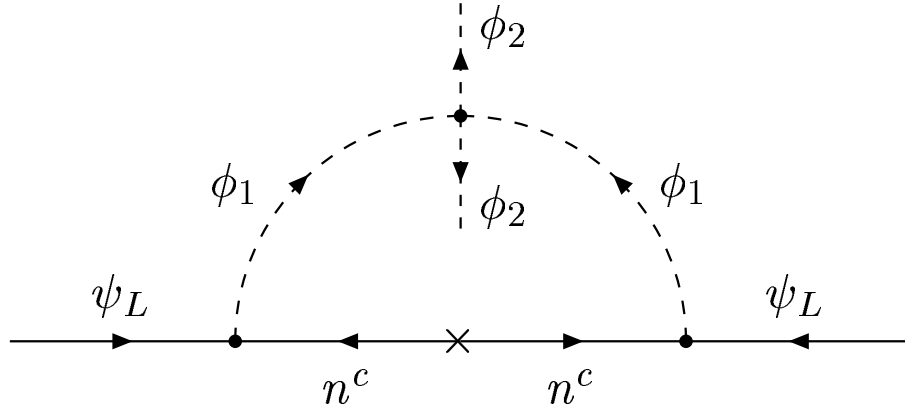

Figure 3: One-loop contribution to $(\phi_2^\dagger \Psi_L)(\phi_2^\dagger \Psi_L)$.

In our model the large mixings in the Maki-Nakagawa-Sakata (MNS) matrix [16] are obtained once the couplings λ_ν and the lepton number violating masses M are allowed to be arbitrary 3×3 matrices. An acceptable rate for FCNC processes (mediated at one loop by the TeV singlets) will require the alignment of the neutrino and the charged-lepton Yukawa couplings. If the charged-lepton mass matrix comes from terms $(\lambda_e/\Lambda)\phi_1\phi_2\Psi_L e^c$ [9], the rotations diagonalizing λ_ν and λ_e must coincide to a large extent. Then, the MNS matrix results from the diagonalization of Eq. (16) (generalized to non-diagonal couplings) in the basis where λ_ν is diagonal. Obviously, the allowed misalignment increases for larger values of f (a detailed quantitative analysis will be presented elsewhere [17]).

The model that we propose provides an explicit realization of TeV-scale non-decoupled neutrinos, which may be observable at a large (e^+e^-) collider [18]. The fields N have a mixing $\approx v/f$ with the SM neutrinos that must be smaller than 0.07 to be consistent with current data [19]. For N masses of order TeV, single heavy neutrino production $e^+e^- \to N\nu \to eW\nu$ could give a signal at CLIC if $v/f \geq 0.005$ [20].

Summary and discussion. In LH models the EW scale is protected from large quadratic corrections only at the one-loop level. Therefore, the framework can not naturally accommodate physics at scales larger than 10 TeV. Neutrino masses are then a puzzle, because in order to explain their size the effective low energy model must involve a much larger scale or very suppressed Yukawa couplings. To be consistent with a *see-saw* mechanism of neutrino masses, LH models should incorporate *another* mechanism to suppress the SM quadratic corrections at the ultraviolet cutoff $\Lambda \approx 10$ TeV. In that case the role of LH ideas would be *just*, for example, to increase the scale of supersymmetry breaking in one order of magnitude. On the other hand, the possibility of very suppressed Yukawa couplings would imply that the LH model is embedded in a theory with extra dimensions or (its CFT dual) strongly coupled dynamics.

We have found a LH alternative that can explain the small size of neutrino masses with no need for a large scale nor extra dynamics. The lepton sector includes a gauge singlet n^c per generation and has unsuppressed Yukawas, but it is free of one-loop quadratic corrections because of the same global symmetry as in the top-quark sector. This implies another $SU(2)$ singlet N per generation, with n^c and $N' \approx N - v/f\ \nu$ combining into a massive field at the scale f of global symmetry breaking. As long as lepton number is not broken, the SM neutrinos are massless. If L is broken at a small scale $M \approx 10^{-6}$ GeV (a *mirror* scale of the one in the *see-saw* mechanism) then the diagram in Fig. 3 introduces a term $\phi_2^\dagger\phi_2^\dagger\Psi_L\Psi_L$ and the SM neutrinos get the observed masses.

In just the SM with an extra singlet per generation this mechanism would not work:

the small value of the singlet mass M would not prevent the SM neutrinos to combine with the singlets and define Dirac fields of mass $\lambda_\nu v$ similar to the mass of quarks and charged leptons. However, the mechanism is naturally implemented in LH models, where the global symmetries imply new fields and allow at the loop level the necessary couplings.

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